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# Kinematics of Edge Dislocations. I. Involutive Distributions of Local Slip Planes

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The geometry of continuous distributions of dislocations and secondary point defects created by these distributions is considered. Particularly, the dependence of a distribution of dislocations on the existence of secondary point defects is modeled by treating dislocations as those located in a time-dependent Riemannian material space describing, in a continuous limit, the influence of these point defects on metric properties of a crystal structure. The notions of local glide systems and involutive distributions of local slip planes are introduced in order to describe, in terms of differential geometry, some aspects of the kinematics of the motion of edge dislocations. The analysis leads, among others, to the definition of a class of distributions of dislocations with a distinguished involutive distribution of local slip planes and such that a formula of mesoscale character describing the influence of edge dislocations on the mean curvature of glide surfaces is valid.

## 1. INTRODUCTION

Let us begin with some general remarks concerning different methods of the description of plastic phenomena [see, e.g., Yang and Lee (1993) for a comprehensive treatment of the subject].

The basic practice in the continuum mechanics approach relies on *a priori*, empirical assumptions for different material responses, under the help of general guidelines deduced axiomatically from basic postulates and limited empirical testing data. In particular, if we deal with the material response to plastic deformations, this approach is termed the mathematical theory of plasticity, or simply *macroplasticity* (e.g., Thomas, 1991; Ivlev, 1966; Perzyna, 1978). One of the consequences of this hypothetical approach lies in the inability to distinguish features presented by a variety of plastic deformation

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mechanisms, traced back at the microstructural level from dislocation glide, phase transformation, to microcracking damage (Yang and Lee, 1993).

The physical theory of plasticity, or simply *microplasticity* (e.g., Suzuki *et al.*, 1991; Hull and Bacon, 1994), deals with the physical background of materials subject to plastic deformation mechanisms and their interrelationship with the evolution of microstructure. However, in practice, microplasticity is rather a set of mathematically simplified (but useful in practical applications) methods for the representation of microscopic data for plastically deformed dislocated crystals than a general theory of microscopic states of such crystals.

So-called *mesoplasticity* is a link between micro- and macro-scales based on the application of microplasticity concepts to various intermediate (or meso) scales where the quantitative theory of continuum mechanics is still applicable in describing the evolution of material structures during the plastic deformation process (e.g., Yang and Lee, 1993; Theodosiu, 1976; Aifantis, 1987). In the mesoplasticity approach, plastic deformation can be, at least in principle, predicted by an Orowan-type theoretical model, i.e., by a generalization of the *Orowan kinematic relation* (verified on a microscopic scale) (Hull and Bacon, 1984):

$$\dot{\gamma} = \rho b v \tag{1.1}$$

where $\dot{\gamma}$ denotes the macroscopic strain rate, $b$ is the modulus of the Burgers vector, $\rho$ is the density of mobile dislocations, and $v$ is the mean dislocation velocity. Therefore, in the mesoplasticity approach, quantities appear that are measured from a direct (microscopic) account of experimental data such as dislocation densities.

Another theory, based on the mesoscale approximation of dislocated crystalline solids, but not relying on the validity of the Orowan-type relations, is the so-called *geometric theory of dislocations* (e.g., Kondo, 1955; Bilby *et al.*, 1958; Kröner, 1986; Trzęsowski, 1993). A continuously dislocated crystal may be described, in the framework of this theory, as a locally homogeneous body endowed with a non-Euclidean geometry representing its material structure in a manner analogous to that used in the gauge theory (Trzęsowski, 1993). The response of a dislocated crystalline solid on its total distortion is then deduced from field equations based on the local invariance of the dislocated crystalline structure (e.g., Trzęsowski, 1993; Kadić and Edelen, 1983) or taking advantage of the existence of the global invariance of geometric objects describing this structure (Trzęsowski and Sławianowski, 1990). However, the response so defined differs from that considered in macroplasticity. Moreover, although the gauge theory (or the theory based on global invariance) may be useful in the description of internal stresses and couple stresses caused by the self-interaction of immobile dislocations

(Trzęsowski 1993; Trzęsowski and Sławianowski, 1990), this theory is not useful for the description of irreversibility of the dislocation motion process and, more generally, for the description of the evolution of the dislocation state under plastic deformation (Kröner, 1995).

However, the physical meaning of the geometric gauge-type theory of dislocations goes beyond its version dealing with the response of a dislocated crystalline solid (Trzęsowski, 1993). It is shown in this paper that, based on a purely geometric version of this theory and taking advantage of the existence of tensorial as well as scalar densities of dislocations (Trzęsowski, 1994, and Section 2), we can derive rigorously Orowan-type formulas (Trzęsowski, 1998). A key to this deduction is the notion of involutive distributions of local slip planes (Section 3). The geometry of these distributions is discussed here and their kinematics is considered in Trzęsowski (1998).

## 2. CONTINUOUS DISTRIBUTIONS OF DISLOCATIONS

It is well known that the occurrence of many dislocations causes the appearance of a particular inelastic distortion of a crystal lattice. Namely, though this distortion breaks the long-range order of a crystalline solid, nevertheless its short-range order is remarkably preserved, and the dislocated crystalline solid can be locally approximately described as a macroscopically small part of an ideal crystal. In the geometric theory of continuized dislocated Bravais crystals (e.g., Kröner, 1986; Trzęsowski, 1993) this inelastic distortion can be introduced by means of a distinguished anholonomic moving frame $\Phi = (\mathbf{E}_a;\ a = 1, 2, 3)$ called the *Bravais moving frame* (Trzęsowski, 1993). If $X = (X^A)$ is a Lagrange coordinate system such that $[X^A] = cm$, $A = 1, 2, 3$, then

$$\mathbf{E}_a = \underset{a}{e}^A \partial_A, \qquad \underset{a}{e}^A \in C^\infty, \qquad \partial_A = \partial/\partial X^A$$

$$[\mathbf{E}_a] = [\partial_A] = \mathrm{cm}^{-1}, \qquad [\underset{a}{e}^A] = [1] \tag{2.1}$$

and the moving coframe $\Phi^* = (E^a;\ a = 1, 2, 3)$ dual to $\Phi$ has the following representation:

$$E^a = \overset{a}{e}_A\, dX^A, \qquad [E^a] = [dX^A] = \mathrm{cm}$$

$$\langle E^a, \mathbf{E}_b\rangle = \overset{a}{e}_A\, \underset{b}{e}^A = \delta^a_b \tag{2.2}$$

The *short-range order* of the dislocated Bravais crystal is represented then by *local crystallographic directions* $L_a = \{s\mathbf{E}_a\colon s \in R\}$, $a = 1, 2, 3$, and its *long-range distortion* is represented by the object of anholonomity ($C^a_{bc}$) of the Bravais moving frame:

$$[\mathbf{E}_a, \mathbf{E}_b] = C^c_{ab}\mathbf{E}_c, \qquad C^c_{ab} \in C^\infty, \qquad [C^c_{ab}] = \mathrm{cm}^{-1} \tag{2.3}$$

where $[\mathbf{u}, \mathbf{v}] = \mathbf{u} \circ \mathbf{v} - \mathbf{v} \circ \mathbf{u}$ denotes the commutator product (bracket) of vector fields $\mathbf{u}$ and $\mathbf{v}$ considered [according to (2.1)] as first-order differential operators. The vanishing of the object of anholonomity means the lack of dislocations. The long-range distortion can be equivalently represented by the so-called *Burgers field* $\tau_\Phi = (\tau^a)$, a triple of 2-forms defined as

$$\tau^a = dE^a = \tfrac{1}{2}\tau^a_{bc}E^b \wedge E^c, \qquad [\tau^a] = \mathrm{cm}, \qquad [\tau^a_{bc}] = \mathrm{cm}^{-1} \tag{2.4}$$

where $\wedge$ denotes the exterior product. The Burgers field is related to the object of anholonomity according to (Yano, 1958)

$$\tau^a_{bc} = -C^a_{bc} \tag{2.5}$$

It is known that the occurrence of many dislocations in a crystalline solid is accompanied by the appearance of point defects. It is, e.g., due to intersections of the dislocation lines. For example, point defects can appear in crossover points of edge dislocations or when two parallel dislocation lines are joining together (Oding, 1961). On the other hand, it is known also that dislocations have no influence on local metric properties of a crystal structure. Consequently, a continuized dislocated Bravais crystal can be endowed with a $\Phi$-parallel *intrinsic metric*, say such that the Bravais moving frame $\Phi$ is orthonormal with respect to it (Kröner, 1986; Trzęsowski, 1994):

$$\begin{aligned} \mathbf{g} &= \delta_{ab}E^a \otimes E^b = g_{AB}\, dX^A \otimes dX^B \\ g_{AB} &= \overset{a}{e}_A \overset{b}{e}_B \delta_{ab}, \qquad [g_{AB}] = [\delta_{ab}] = [1], \qquad [\mathbf{g}] = \mathrm{cm}^2 \end{aligned} \tag{2.6}$$

with the corresponding *material volume* 3-form of the form

$$\begin{aligned} V = E^1 \wedge E^2 \wedge E^3 &= \tfrac{1}{6} e_{ABC}\, dX^A \wedge dX^B \wedge dX^C \\ &= e\, dX^1 \wedge dX^2 \wedge dX^3, \qquad [V] = \mathrm{cm}^3 \end{aligned} \tag{2.7}$$

where

$$e_{ABC} = e\epsilon_{ABC}, \qquad e = \det(\overset{a}{e}_A) = g^{1/2}, \qquad g = \det(g_{AB}),\ [e] = [1] \tag{2.8}$$

where $\epsilon_{ABC}$ denotes the permutation symbol associated with the cobase fields $dX^A$, $A = 1, 2, 3$. Since the metric tensor (2.6) is flat if dislocations are absent (i.e., $\tau^a = 0$, $a = 1, 2, 3$), this metric tensor may be interpreted as the one representing, in a continuous limit, the influence of *secondary point defects* (i.e., created by the considered distribution of dislocations) on metric properties of the Bravais crystal (Trzęsowski, 1994). Consequently, we can consider the base vector fields of a Bravais moving frame as those defining the Burgers field as well as the local scales of an *internal length measurement* along local crystallographic directions of the continuized dislocated Bravais crystal. It ought to be stressed that these base vector fields do not describe translational symmetries of local ideal Bravais lattices. This is because translational symmetries of a crystal structure are lost in the continuous limit defining the continuized Bravais crystal [however, rotational symmetries of this crystal structure are locally preserved (Trzęsowski, 1993)]. Finally, we see that the triple $(\Phi, \mathbf{g}, \tau_\Phi)$ defines a continuous geometric model of the inelastic distortion of a Bravais crystal structure due to the occurrence of many dislocations (Trzęsowski, 1994).

The long-range distortion of a crystal structure due to dislocations can be quantitatively measured by the *Burgers vector.* Its local counterpart, called the local Burgers vector, can be introduced in the following way [see Trzęsowski (1994) for a more precise reasoning]. According to the above representation of the influence of many dislocations on a Bravais crystal structure, the dependence of a distribution of dislocations on the existence of secondary point defects may be modeled by treating dislocations as those located in the Riemannian *material space* $\mathcal{B}_g = (\mathcal{B}, \mathbf{g})$ describing the distortion of metric properties of the crystal structure (Trzęsowski, 1994); $\mathcal{B}$ denotes the body identified with an open, simply connected subset of the Euclidean point space $E^3$ (the configurational space of the body). Next, if the vector-valued 2-form $\Sigma$ defined by

$$\begin{aligned} \mathbf{\Sigma} &= \mathbf{E}_a \otimes \Sigma^a, \qquad \Sigma^a = \delta^{ab}\Sigma_b \\ \Sigma_a &= \tfrac{1}{2}\, e_{abc}\Sigma^{bc}, \qquad \Sigma^{bc} = E^b \wedge E^c \end{aligned} \tag{2.9}$$

where

$$e_{abc} = g^{1/2}\epsilon_{abc}, \qquad g = \det|(\delta_{ab})| = 1 \tag{2.10}$$

and $\epsilon_{abc}$ denotes the permutation symbol, is considered as the one representing a surface element $d\Sigma$ of a surface $\Sigma \subset \mathcal{B}_g$ (i.e., $\Sigma$ is a 2-dimensional submanifold of $\mathcal{B}_g$) with the unit normal $\mathbf{l}$, i.e., if we identify

$$\begin{aligned} \mathbf{\Sigma} &\cong d\Sigma\mathbf{l}, \qquad \mathbf{l} = l^a\mathbf{E}_a \\ \|\mathbf{l}\|_g^2 &= \delta_{ab}l^a l^b = 1, \qquad [\mathbf{l}] = \mathrm{cm}^{-1} \end{aligned} \tag{2.11}$$

then it follows from (2.4) and (2.9) that the component $\tau^a$ of the Burgers field $\tau_\Phi$ may be identified with an infinitesimal quantity $\delta b^a$ of the dimension of the Burgers vector component (Trzęsowski, 1994):

$$\begin{aligned} \tau^a &\cong \delta b^a, \qquad \delta b^a = \rho d\Sigma b^a \\ [\rho] &= \mathrm{cm}^{-2}, \qquad [b^a] = [\delta b^a] = \mathrm{cm}, \qquad [d\Sigma] = \mathrm{cm}^2 \end{aligned} \tag{2.12}$$

where $\rho$ is a positive scalar independent of the choice of $\mathbf{l}$, and

$$\rho b^a = l_b \alpha^{ba}, \qquad l_a = \delta_{ab} l^b \tag{2.13a}$$

$$\alpha^{ab} = \tfrac{1}{2} e^{acd} \tau^b_{cd}, \qquad [\alpha^{ab}] = \text{cm}^{-1} \tag{2.13b}$$

where $e^{abc} = \epsilon^{abc} = \epsilon_{abc}$. Let us consider $\mathbf{l}$ as the $\mathbf{g}$-unit vector field tangent to a *dislocation line*, understood here (at least locally) as the one defining a boundary between slipped and unslipped parts of the crystal (Hull and Bacon, 1984; Trzęsowski, 1998) normal to the surface element $d\Sigma$. Assuming the scalar $\rho$ to be the volume *scalar density of dislocations* defined as the length of all dislocation lines included in the volume unit of the Riemannian material space $\mathcal{B}_g$, we can define the *local Burgers vector* as $\mathbf{b} = b^a \mathbf{E}_{\hat{a}}$, $[\mathbf{b}] = [1]$. The tensor field $\boldsymbol{\alpha} = \alpha^{ab} \mathbf{E}_a \otimes \mathbf{E}_b$ is called the *dislocation density tensor* (Trzęsowski, 1994). A line in $\mathcal{B}_g$ (with its unit tangent $\mathbf{l}$) is interpreted as the *edge* dislocation line if (Trzęsowski, 1994)

$$b^a l_a = b_g m^a l_a = 0, \qquad b_g > 0 \tag{2.14}$$

where

$$\mathbf{b} = b_g \mathbf{m}, \qquad \mathbf{m} = m^a \mathbf{E}_a, \qquad \|\mathbf{m}\|_g = 1$$

$$b_g = \|\mathbf{b}\|_g = (b_a b^a)^{1/2}, \qquad [b_g] = \text{cm}, \qquad [\mathbf{m}] = \text{cm}^{-1} \tag{2.15}$$

or as the *screw* dislocation line if

$$b^a = \eta l^a, \qquad \eta \neq 0 \tag{2.16}$$

In other cases the line is interpreted as a *mixed* (edge and screw) dislocation line. Note that a line in $\mathcal{B}_g$ may be interpreted as a dislocation line iff $b_g \neq 0$. If $\mathbf{l}$ is tangent to a dislocation line, then the local plane $\pi(\mathbf{l}, \mathbf{b})$ containing vectors $\mathbf{l}$ and $\mathbf{b}$ is interpreted as a *local slip plane* (Trzęsowski, 1994). For an edge dislocation line its local slip planes are univocally defined and normal to the $\mathbf{n}$ direction; the triple $(\mathbf{l}, \mathbf{m}, \mathbf{n})$, where $\mathbf{l}$ is the unit tangent to the line and $\mathbf{m}$ is defined by (2.13)–(2.15), constitutes a $\mathbf{g}$-orthonormal vector base univocally defined (up to its orientation) along the line. For screw dislocation lines the local slip planes are not univocally defined. If $\mathbf{l}$ is an unit vector field on $\mathcal{B}_g$ such that the conditions (2.13)–(2.15) are fulfilled, then the corresponding ordered triple $(\mathbf{l}, \mathbf{m}, \mathbf{n})$ of $\mathbf{g}$-orthonormal vector fields on $\mathcal{B}_g$ will be called a *local glide system* for a family of edge dislocation lines admitted by the considered distribution of dislocations and tangent to $\mathbf{l}$. The pair $(\mathbf{m}, \mathbf{n})$ will be called then the *local slip system* (of the local glide system). A distribution of dislocations such that each dislocation line is edge may be identified with a *distribution of edge dislocations.*

It follows from (2.5) and (2.13b) that the object of anholonomity can be written in terms of the dislocation density tensor:

$$C^a_{bc} = t_{[b}\delta^a_{c]} - e_{bcd}\gamma^{da}$$

$$\gamma^{ab} = \alpha^{(ab)}, \qquad t_a = e_{abc}\alpha^{bc} = C^b_{ab} \tag{2.17}$$

Particularly, the following object of anholonomity defines a distribution of edge dislocations:

$$C^c_{ab} = t_{[a}\delta^c_{b]} = \mu_g n_{[a}\delta^c_{b]} \tag{2.18}$$

where

$$\mathbf{t} = t^a\mathbf{E}_a = \mu_g\mathbf{n}, \qquad t^a = \delta^{ab}t_b, \qquad [t_a] = [\mathbf{n}] = \mathrm{cm}^{-1}$$

$$\mu_g = (t_a t^a)^{1/2} > 0, \qquad n^a n_a = 1, \qquad [\mu_g] = \mathrm{cm}^{-1} \tag{2.19}$$

Namely, the dislocation density tensor $\boldsymbol{\alpha}$ and the local Burgers vector $\mathbf{b}$ are given then by

$$\alpha^{ab} = \tfrac{1}{2}t_c e^{cba} = -\alpha^{ba} \tag{2.20}$$

and

$$\rho b^a = \tfrac{1}{2}t_c l_b e^{cba}, \qquad \rho > 0 \tag{2.21}$$

So, in this case [see (2.15) and (2.19)]

$$m^a l_a = m^a n_a = 0 \tag{2.22a}$$

$$\rho b_g = \tfrac{1}{2}\mu_g[1 - (n^a l_a)]^{1/2} \geq 0 \tag{2.22b}$$

It follows from (2.14) and (2.22a) that each dislocation line is edge. Moreover, it follows from (2.22b) that a line in $\mathcal{B}_g$ is the dislocation line iff $|n^a l_a| \neq 1$. We will consider further dislocation lines such that

$$n^a l_a = 0 \tag{2.23}$$

or, equivalently, such that the formula

$$\rho b_g = \tfrac{1}{2}\mu_g > 0 \tag{2.24}$$

stating that the local Burgers vector modulus $b_g$ is a positive scalar independent of the choice of $\mathbf{l}$, holds. It follows from (2.4), (2.5), and (2.18) that in the case (2.24) we have

$$\tau^a = \rho b_g E^a \wedge n, \qquad n = n_a E^a, \qquad n_a = \delta_{ab}n^b \tag{2.25}$$

and, for the local glide system ($\mathbf{l}$, $\mathbf{m}$, $\mathbf{n}$) defined by the conditions (2.15), (2.19), (2.21), and (2.24), the identification (2.12) with

$$b^a = e^{abc}\beta_{bc}, \qquad \beta_{bc} = b_g n_{[b}l_{c]} \tag{2.26}$$

is valid. The local Burgers vector $\mathbf{b} = b^a\mathbf{E}_a$ may be interpreted, according to (2.12) and (2.26), as a vector field of vortices (of the tensorial density $\beta_{ab}$) in a cylinder tube with the section $d\Sigma$. A Burgers field of the form (2.25) may thus be viewed as the one defining a "*dislocation fluid*" (of the scalar density $\rho$) consisting of infinitesimal edge dislocation loops (Trzęsowski, 1997). For example, the irradiation of a crystal with fast neutrons produces very small circular edge dislocation loops (Bullough and Newman, 1970). The loops can be treated then (in the continuized crystal approximation) as infinitesimal ones. Note that if the covector field $n$ of (2.25) is interpreted as representing a certain class of pairs of parallel planes with equal distances (Schouten, 1954), then (2.12), (2.25), and (2.26) mean that the infinitesimal loops as well as their infinitesimal Burgers vectors $\delta\mathbf{b} = \delta b^a\mathbf{E}_a$ "lie" in parallel planes defined by $n$. On the other hand, (2.21), (2.22a), and (2.23) mean that each plane belonging to this family of planes is a local slip plane. Thus, one has both a glide motion of infinitesimal edge dislocation loops (lying on the same local slip plane) and a local double cross-slip process in which the Burgers vector is parallel to a slip plane, but the dislocation line is bent in such a way that one part lies on the slip plane and the other on the plane parallel to it (Hull and Bacon, 1984).

Let $\Phi(t) = (\mathbf{E}_a(\cdot, t);\ a = 1, 2, 3,\ t \in I \subset R_+)$, be a time-dependent (with the time treated as a parameter) Bravais moving frame, and let $\Phi^*(t) = (E^a(\cdot,t))$ denote the Bravais moving coframe dual to $\Phi(t)$ [see (2.1) and (2.2)]. The corresponding Burgers field $\tau_\Phi(t) = (\tau^a(\cdot, t))$ is then a triple of 2-forms depending on the time parameter and defined, according to (2.4), by spatial external derivatives of 1-forms $E^a(\cdot, t)$. If $\mathbf{g}(\cdot, t)$ is the corresponding intrinsic metric, then we will denote by $\mathbf{g}_t$ an *instantaneous intrinsic metric* defined by

$$\begin{aligned}\mathbf{g}_t(X) = \mathbf{g}(X, t) &= \delta_{ab}E^a(X, t) \otimes E^b(X, t)\\ &= g_{AB}(X, t)dX^A \otimes dX^B \qquad (2.27)\end{aligned}$$

and we will denote by $\mathcal{B}_t = (\mathcal{B},\mathbf{g}_t)$ the Riemannian *instantaneous material space*. In (2.8) we have then

$$e_{ABC} = e_{ABC}(X, t) = e(X, t)\epsilon_{ABC}$$

$$e = e(X, t) = \det(\overset{a}{e}_A(X, t)), \qquad g = g(X, t) = \det(g_{AB}(X, t)) \quad (2.28)$$

## 3. DISTRIBUTIONS OF LOCAL SLIP PLANES

Let us consider a distinguished family $\pi = \{\pi_p,\ p \in \mathcal{B}\}$ of local slip planes (Section 2) defining the so-called two-dimensional distribution on $\mathcal{B}_g$.

A two-dimensional *distribution* (of planes) is called (completely) *integrable* if there exists a family $\Sigma = \{\Sigma_p, p \in \mathcal{B}\}$ of two-dimensional submanifolds of $\mathcal{B}_g$, called integral manifolds of $\pi$, such that $p \in \Sigma_p$ and for each $q \in \Sigma_p$ the plane $\pi_q$, is tangent to $\Sigma_p$ in $q$. For example, the local glide system $(\mathbf{l}, \mathbf{m}, \mathbf{n})$ defined by (2.15), (2.19), (2.21), and (2.24) defines a two-dimensional integrable distribution (of local slip planes) if

$$dt = 0, \qquad t = t_a E^a \tag{3.1}$$

that is, at least locally, we have then

$$t = d\varphi \tag{3.2}$$

If the first the de Rham cohomology class of the three-dimensional manifold $\mathcal{B}_g$ vanishes (e.g., it is the case of a three-dimensional affine space) or, more generally, if the manifold is contractible to a point, then the potential $\varphi$ of (3.2) is defined globally.

Let $\mathbf{E}_\alpha(p) \in \pi_p$, $\alpha = 1, 2$, be a base of the two-dimensional vector-space $\pi_p$. The distribution $\pi$ is called *involutive* iff there are $C^\infty$-functions $C^\kappa_{\alpha\beta}$, $\alpha$, $\beta$, $\kappa = 1, 2$, on $\mathcal{B}_g$ such that (Sikorski, 1972)

$$[\mathbf{E}_\alpha, \mathbf{E}_\beta] = C^\kappa_{\alpha\beta}\mathbf{E}_\kappa \tag{3.3}$$

A distribution is (completely) integrable iff it is involutive (Sikorski, 1972). If the condition (3.3) is fulfilled, the system of equations [see (2.1)]

$$\mathbf{E}_\alpha\varphi = \underset{\alpha}{e}^A\partial_A\varphi = 0, \qquad A = 1, 2, 3; \quad \alpha = 1, 2 \tag{3.4}$$

has a solution that defines surfaces of the family $\Sigma$ of integral manifolds as those given by

$$\Sigma_c\colon \quad \varphi(X) = c, \qquad d\varphi \neq 0 \tag{3.5}$$

where $c \in R$ is a constant, i.e., $\Sigma = \{\Sigma_c, c \in R\}$ (Sinukov, 1979). It can be shown (Von Westenholz, 1978) that for each $p \in \mathcal{B}$ there are then coordinates $X = (X^A; A = 1, 2, 3)$ at $p$ such that $X^3 = \varphi$. For any such coordinates $\partial_\alpha = \partial/\partial X^\alpha$, $\alpha = 1, 2$, is a local basis for $\Sigma$ and the slices

$$\Sigma_c = \{q \in U\colon X^3(q) = c\} \tag{3.6}$$

where $U$ is a coordinate neighborhood of $p$, belong to $\Sigma$. Consequently, at least locally, we can consider an involutive distribution as the one that integral manifolds $\Sigma_c$ are defined by a *Lagrange coordinate system* on $\mathcal{B}$. Moreover, it is known (Von Westenholz, 1978) that $\mathcal{B}_g$ is *foliated* by the distribution $\pi$, that is, through each point $p \in \mathcal{B}_g$ there passes an unique maximal integral manifold of $\pi$. Since the considered distribution $\pi$ consists of local slip planes, these integral manifolds will be called *slip surfaces.* If the distribution

$\pi$ is defined by a local glide system, the slip surfaces are virtually surfaces of a glide motion of edge dislocation lines, and then these surfaces will be called also (virtual) *glide surfaces.*

Let $\mathcal{B}_t = (\mathcal{B}, \mathbf{g}_t)$ denote the Riemannian instantaneous material space (Section 2). The formulas (3.3) and (3.6) suggest that we consider a time-dependent Bravais moving frame $\Phi(t) = (\mathbf{E}_a(\cdot, t))$ such that for each $t \in I$ there exists a coordinate system $X = (X^A) = (X^\kappa, X^3)$ on $\mathcal{B}_t$ in which

$$\mathbf{E}_3(X, t) \doteq \partial_3 = \partial/\partial X_3$$

$$\mathbf{E}_\alpha(X, t) \doteq \Psi_t^{-1/2}(X^3)\mathbf{e}_\alpha(X^\kappa, t), \qquad \alpha, \kappa = 1, 2 \tag{3.7}$$

where $\doteq$ means that the formulas of (3.7) are valid in a distinguished coordinate system on $\mathcal{B}_t$. For example, if $\xi = (\xi^a;\ a = 1, 2, 3)$ is a reference coordinate system defined in $\mathcal{B}_0$, then the coordinate transformation

$$X^\alpha = \chi^\alpha(\xi^\kappa, t), \qquad \chi^\alpha(\xi^\kappa, 0) = \xi^\alpha, \qquad X^3 = \xi^3, \quad \alpha, \kappa = 1, 2 \tag{3.8}$$

defines a *convective Lagrange coordinate system* on $\mathcal{B}$ preserving, at each instant $t \in I$, the form (3.7) of the Bravais moving frame. Convective Lagrange coordinate systems will be considered as those defined on the time-dependent Riemannian space $\mathcal{B}_g$. Denoting

$$[\mathbf{e}_\alpha, \mathbf{e}_\beta] = c^{\kappa}_{\alpha\beta}\mathbf{e}_\kappa \tag{3.9}$$

we obtain the condition (3.3) with

$$C^{\kappa}_{\alpha\beta}(X, t) = \Psi_t^{-1}(X^3)c^{\kappa}_{\alpha\beta}(X^\omega, t) \tag{3.10}$$

Therefore, a Bravais moving frame defined by the condition (3.7) defines an involutive distribution $\pi$ consisting of local planes spanned by the base vector fields $\mathbf{E}_\alpha$, $\alpha = 1, 2$.

The object of anholonomity $(C^c_{ab})$ [see (2.3)] for the Bravais moving frame of the form (3.7) is defined by (3.10) and

$$C^{\kappa}_{\alpha 3}(X, t) = -(\Psi_t'/\Psi_t)(X^3)\delta^3_{[\alpha}\delta^\kappa_{3]}$$

$$C^3_{\alpha\beta}(X, t) = 0, \qquad C^3_{\alpha 3}(X, t) = 0 \tag{3.11}$$

where denoted $\Psi_t' = \partial_3\Psi_t$. Particularly, we have

$$[\mathbf{E}_\alpha, \mathbf{E}_3] = \tfrac{1}{2}(\Psi_t'/\Psi_t)\mathbf{E}_\alpha \tag{3.12}$$

Introducing designations [see (2.17)]

$$t_a = C^c_{ac} = C^\kappa_{a\kappa}$$

$$t_\alpha = C^\kappa_{\alpha\kappa} = \Psi_t^{-1/2}c^\kappa_{\alpha\kappa}$$

$$[\mathbf{e}_1, \mathbf{e}_2] = \alpha_1\mathbf{e}_1 + \alpha_2\mathbf{e}_2 \tag{3.13}$$

we obtain

$$t_1 = \Psi_t^{-1/2}\alpha_2, \qquad t_2 = -\Psi_t^{-1/2}\alpha_1, \qquad t_3 = \Psi_t'/\Psi_t \tag{3.14}$$

and

$$C^c_{\alpha\beta} = 2t_{[\alpha}\delta^c_{\beta]}, \qquad C^c_{\alpha 3} = t_3\delta^c_{[\alpha}\delta^c_{3]} \tag{3.15}$$

where $\alpha, \beta = 1, 2, c = 1, 2, 3$. The components $\alpha^{ab}$ of the dislocation density tensor defined by (2.5), (2.13b), (2.17), and (3.11)–(3.15) take the following form in the coordinate system of (3.7):

$$\begin{pmatrix} & a \downarrow 1, 2, 3 \\ \alpha^{ab}; & \\ & b \rightarrow 1, 2, 3 \end{pmatrix} = \begin{pmatrix} 0 & t_3/2 & 0 \\ -t_3/2 & 0 & 0 \\ t_2 & -t_1 & 0 \end{pmatrix} \tag{3.16}$$

and the components $b^a$, $a = 1, 2, 3$, of the local Burgers vector defined by (2.13a) are given by

$$\rho b^\alpha \doteq (l_3 t_\kappa - \tfrac{1}{2} t_3 l_\kappa)e^{3\alpha\kappa}, \qquad \rho b^3 = 0 \tag{3.17}$$

Consequently,

$$\rho b^a l_a = \rho b^\alpha l_\alpha = l_3(t_1 l_2 - t_2 l_1) \tag{3.18}$$

Thus, a dislocation line is an edge dislocation line [i.e., the condition (2.14) is fulfilled] iff $l_3 = 0$ or $t_1 l_2 = t_2 l_1$. Note that there are no screw dislocation lines [the condition (2.16)]. Since, according to (3.17) and (3.18), for an edge dislocation line such that

$$l_3 = \mathbf{l} \cdot \mathbf{E}_3 = 0 \tag{3.19}$$

where $\mathbf{a} \cdot \mathbf{b} = \mathbf{agb}$, we have

$$b_3 = \mathbf{b} \cdot \mathbf{E}_3 = 0 \tag{3.20}$$

the local slip planes $\pi$ $(\mathbf{l}, \mathbf{b})$ are normal to the $\mathbf{n}$ direction defined as

$$\mathbf{n} = \mathbf{E}_3 \tag{3.21}$$

Thus, the planes spanned by the base vectors $\mathbf{E}_\alpha$, $\alpha = 1, 2$, cover the local slip planes $\pi(\mathbf{l}, \mathbf{b})$, and the maximal integral manifolds of the involutive distribution so defined are (virtual) glide surfaces for edge dislocation lines defined by (3.19). The corresponding local glide system $(\mathbf{l}, \mathbf{m}, \mathbf{n})$ is defined by the conditions (3.19)–(3.21) with [see (2.15)]

$$b^\alpha = b_g m^\alpha, \qquad m^3 = 0 \tag{3.22a}$$

$$2\rho b_g = |t_3| = |\Psi_t'/\Psi_t| > 0 \tag{3.22b}$$

The condition (3.22b) means that the local Burgers vector modulus $b_g$ is a positive scalar independent of the choice of **l** [cf. the condition (2.24)].

The intrinsic metric tensor **g** takes the following form in a coordinate system of (3.7) [e.g., defined by the convective Lagrange coordinate system (3.8)]:

$$\mathbf{g}(X, t) = \mathbf{g}_t(X) \doteq \Psi_t(X^3)\mathbf{a}_t(X^\kappa) + dX^3 \otimes dX^3 \tag{3.23}$$

where $\mathbf{a}_t$ is the metric tensor of a general 2-dimensional Riemannian space represented, according to (2.27) and (3.7), in the form

$$\mathbf{a}_t(X^\kappa) = \delta_{\alpha\beta} e^\alpha(X^\kappa, t) \otimes e^\beta(X^\kappa, t), \qquad \langle e^\alpha, \mathbf{e}_\beta \rangle = \delta^\alpha_\beta, \qquad \alpha, \beta, \kappa = 1, 2 \tag{3.24}$$

## 4. EQUIDISTANT MATERIAL SPACE

The form (3.23) of the intrinsic metric tensor covers the canonical form of a metric tensor of the so-called *equidistant* Riemannian space (Sinukov, 1979). Namely, this Riemannian space is defined by the following conditions:

$$\begin{gathered} \nabla^g_A \varphi_B = \zeta g_{AB}, \qquad \zeta \neq 0 \\ \varphi_A = \varphi_g n_A, \qquad n_A n^A = 1, \qquad \varphi_g > 0 \end{gathered} \tag{4.1}$$

where $\nabla^g = (\Gamma^A_{BC}\,[\mathbf{g}])$ denotes the Levi-Civita covariant derivative with its Christoffel symbols $\Gamma^A_{BC}[\mathbf{g}]$ based on the metric tensor **g**. It follows from (4.1) that

$$\varphi_A = \partial_A \varphi \tag{4.2a}$$

$$n^A \nabla^g_A n_B = 0, \qquad n^A = g^{AB} n_B \tag{4.2b}$$

where $\varphi$ is a smooth scalar. If **g** is a time-dependent intrinsic metric, the conditions (4.1) and (4.2) would be considered for the instantaneous intrinsic metrics $\mathbf{g}_t$, $t \in I$. The condition (4.2a) defines, then, for each instantaneous material space $\mathcal{B}_t = (\mathcal{B}, \mathbf{g}_t)$ a family $\Sigma_t = \{\Sigma_{c,t} = \varphi_t^{-1}(c),\ c \in R\}$ of surfaces orthogonal to the geodesic congruence (in the sense of $\mathbf{g}_t$) of curves tangent to the unit vector field $\mathbf{n} = \mathbf{n}\,(X, t)$. Each surface $\Sigma_{c,t}$ belonging to this family, considered as a 2-dimensional submanifold of $\mathcal{B}_t$, is *umbilical* with the constant mean curvature $H_t(c)$ (Schouten, 1954), and can be characterized as made up of endpoints of the geodesics of the same length starting, e.g., from the surface $\Sigma_{0,t} = \varphi_t^{-1}(0)$ (Sinukov, 1979). This is the *equidistant property* of the normal congruence defined by the vector field $\boldsymbol{\varphi} = \varphi_g \mathbf{n}$. It can be shown (Sinukov, 1979) that for each $t \in I$ there exists a coordinate system $X = (X^\kappa, X^3)$ on $\mathcal{B}_t$ such that $X^3 = \varphi_t$,

$$n^A = \delta^A_3 \tag{4.3}$$

and the surfaces $\Sigma_{c,t}$ have, at least locally, a coordinate description of the form (3.6). Moreover, in this coordinate system, the instantaneous intrinsic metric $\mathbf{g}_t$ has the representation (3.23) with

$$\Psi_t(X^3) = a(t)^2 \exp[-2\kappa_t(X^3)] \tag{4.4a}$$

$$\kappa_t(X^3) = -\int \zeta(X^3, t)\, dX^3 \tag{4.4b}$$

Assuming that $\Psi_t(0) = 1$, we obtain

$$\Psi_t(X^3) = \exp\{-2[\kappa_t(X^3) - \alpha(t)]\} \tag{4.5a}$$

$$a(t) = \exp[\alpha(t)], \qquad \alpha(t) = \kappa_t(0) \tag{4.5b}$$

A Lagrange coordinate system defined by the conditions (3.6), (3.23), (4.3), and (4.5) will be called *actual*. The existence of actual coordinates suggests that we consider a class of convective Lagrange coordinate systems $X^A = \chi^A(\xi, t)$, where $\xi = (\xi^a)$ is a reference coordinate system on $\mathcal{B}_0$, such that $X = (X^A)$ is an actual coordinate system at the instant $t \in I$. The convective Lagrange coordinate systems so defined generalize those introduced by the coordinate transformation (3.8).

A metric tensor $\mathbf{a}_{c,t}$, induced on the coordinate surface $\Sigma_c$ of an actual coordinate system [see (3.6) and (3.23)], depends on the parameter $t$ explicitly and defines the first fundamental form of the 2-dimensional submanifold $\Sigma_{c,t} = (\Sigma_c, \mathbf{a}_{c,t})$ of the instantaneous equidistant Riemannian space $\mathcal{B}_t$:

$$\begin{aligned}
\mathbf{a}_{c,t}(X^\kappa) &= \Psi_t(c)\mathbf{a}_t(X^\kappa) = a[c, t]_{\alpha\beta}(X^\kappa)\, dX^\alpha \otimes dX^\beta \\
\mathbf{a}_t(X^\kappa) &= \mathbf{a}(X^\kappa, t) = a_{\alpha\beta}(X^\kappa, t)\, dX^\alpha \otimes dX^\beta \\
a[c, t]_{\alpha\beta}(X^\kappa) &= \Psi_t(c) a_{\alpha\beta}(X^\kappa, t), \qquad \Psi_t(0) = 1
\end{aligned} \tag{4.6}$$

Since the surface $\Sigma_{c,t} \subset \mathcal{B}_t$ is umbilical, its second fundamental form $\mathbf{b}_{c,t}$ has the following form (Eisenhart, 1964):

$$\begin{aligned}
\mathbf{b}_{c,t}(X^\kappa) &= b[c, t]_{\alpha\beta}(X^\kappa)\, dX^\alpha \otimes dX^\beta \\
b[c, t]_{\alpha\beta}(X^\kappa) &= \frac{H_t(c)}{2} a[c, t]_{\alpha\beta}(X^\kappa)
\end{aligned} \tag{4.7}$$

where the mean curvature $H_t(c)$ of $\Sigma_{c,t}$ has the form

$$H_t(c) = 2\kappa_t'(c) \tag{4.8}$$

where $\kappa_t$ is defined by (4.4b) and $\kappa_t' = \partial_3 \kappa_t$. The mean curvature of $\Sigma_{0,t}$ will be denoted by $H_a(t)$:

$$H_a(t) = H_t(0) = 2\kappa_t'(0) \tag{4.9}$$

We conclude that if the scalar $\Psi_t$ of (3.7) has the form (4.5), then the time-dependent local glide system (**l**, **m**, **n**) defined by (3.19)–(3.21) describes a foliation of the (time-dependent) equidistant material space $\mathcal{B}_g = (\mathcal{B}, \mathbf{g})$ by umbilical glide surfaces (see Section 3) with the constant mean curvature of the form (4.8). Moreover, it follows from (2.15),(3.22b), (4.5), and (4.8) that along each glide surface $\Sigma_{c,t}$, the formula

$$\rho b_{g|\Sigma_{c,t}} = \tfrac{1}{2}\,|H_t(c)| \tag{4.10}$$

describing the influence of edge dislocations on the mean curvature of the glide surface is valid.

The Christoffel symbols $\Gamma^A_{BC}[\mathbf{g}]$ of the Levi-Civita covariant derivative $\nabla^g$ take, in coordinates $X = (X^\kappa, X^3)$ of (3.23) with $\Psi_t$ given by (4.5), the following form (Yano, 1958; Sinukov, 1979):

$$\begin{aligned}
&\Gamma^3_{33}[\mathbf{g}_t] = \Gamma^3_{3\alpha}[\mathbf{g}_t] = \Gamma^\alpha_{33}[\mathbf{g}_t] = 0\\
&\Gamma^\alpha_{\beta 3}[\mathbf{g}_t] = \tfrac{1}{2}(\Psi_t'/\Psi_t)\delta^\alpha_\beta, \qquad \Psi_t' = 2\kappa_t'\Psi_t\\
&\Gamma^\kappa_{\alpha\beta}[\mathbf{g}_t] = \Gamma^\kappa_{\alpha\beta}[\mathbf{a}_t], \qquad \Gamma^3_{\alpha\beta}[\mathbf{g}_t] = -\tfrac{1}{2}\Psi_t' a_{\alpha\beta}
\end{aligned} \tag{4.11}$$

where $f' = \partial_3 f$, $\nabla^a = (\Gamma^\kappa_{\alpha\beta}[\mathbf{a}_t])$ denotes the Levi-Civita covariant derivative on the two-dimensional manifold $\Sigma_{0,t} = (\Sigma_0, \mathbf{a}_t)$, and we took into account that $\Gamma^\kappa_{\alpha\beta}[\mathbf{a}_{c,t}] = \Gamma^\kappa_{\alpha\beta}[\mathbf{a}_c]$. It follows from (4.11) that the nonvanishing components $R^D_{ABC} = R^D_{ABC}\,[\mathbf{g}]$ of the covariant derivative $\nabla^g$ are, at the instant $t \in I$, given by (Yano, 1958)

$$\begin{aligned}
&R^{\,3}_{\alpha 3\gamma} = -R^3_{3\alpha\gamma} = \tfrac{1}{2}\,[\Psi_t''/\Psi_t - \tfrac{1}{2}(\Psi_t'/\Psi_t)^2]g_{\alpha\gamma}\\
&R^{\,\kappa}_{\alpha 33} = -R^\kappa_{3\alpha 3} = \tfrac{1}{2}\,[\Psi_t''/\Psi_t - \tfrac{1}{2}(\Psi_t'/\Psi_t)^2]\delta^\kappa_\alpha\\
&R^{\,\kappa}_{\alpha\beta\gamma} = -R^\kappa_{\beta\alpha\gamma} = R^{\,\kappa}_{\alpha\beta\gamma}[\mathbf{a}_t] - \tfrac{1}{4}(\Psi_t'/\Psi_t)^2(\delta^\kappa_\alpha g_{\beta\gamma} - \delta^\kappa_\beta g_{\alpha\gamma})
\end{aligned} \tag{4.12}$$

where $R^\kappa_{\alpha\beta\gamma}[\mathbf{a}_t]$ are components of the curvature tensor of the covariant derivative $\nabla^a$. The components $R_{AB} = R^C_{CAB}$ of the Ricci tensor of the covariant derivative $\nabla^g$ have, at the instant $t \in I$, the following form:

$$\begin{aligned}
&R_{\alpha 3} = 0, \qquad g_{\alpha 3} = 0\\
&R_{33} = -[\Psi_t''/\Psi_t - \tfrac{1}{2}(\Psi_t'/\Psi_t)^2]g_{33}, \qquad g_{33} = 1\\
&R_{\beta\gamma} = R^3_{3\beta\gamma} + R^\alpha_{\alpha\beta\gamma} = R_{\beta\gamma}[\mathbf{a}_t] - \tfrac{1}{2}(\Psi_t''/\Psi_t)g_{\beta\gamma} = Ag_{\beta\gamma}
\end{aligned} \tag{4.13}$$

where $R_{\alpha\beta}[\mathbf{a}_t]$ are components of the Ricci tensor of the covariant derivative $\nabla^a$, and

$$\begin{aligned}
&R_{\alpha\beta}[\mathbf{a}_t] = K_a(t)a_{\alpha\beta} = K_t g_{\alpha\beta}\\
&K_a(t) = \tfrac{1}{2}\,a^{\alpha\beta}R_{\alpha\beta}[\mathbf{a}_t], \qquad K_t = K_a(t)/\Psi_t\\
&A = K_t - \tfrac{1}{2}(\Psi_t''/\Psi_t), \qquad g_{\alpha\beta} = \Psi_t a_{\alpha\beta}, \qquad \Psi_t(0) = 1
\end{aligned} \tag{4.14}$$

The scalar curvature $K_g$ of the equidistant space $\mathcal{B}_g$ is given by

$$K_g = \tfrac{1}{6} g^{AB} R_{AB} = \tfrac{1}{3}(2\kappa_t'' - K_t) - \kappa_t'^2 \tag{4.15}$$

If the three-dimensional material space $\mathcal{B}_g$ is an *Einstein space*, that is,

$$R_{AB} = 2K_g g_{AB} \tag{4.16}$$

and thus $\mathcal{B}_g$ is, at each instant $t \in I$, a conformally flat space of a constant scalar curvature $K_g = K_g(t)$ (Eisenhart, 1964), then the instantaneous umbilical surface $\Sigma_{c,t} \subset \mathcal{B}_t$ has a constant scalar curvature $K_t(c)$ of the form (Eisenhart, 1964)

$$K_t(c) = \tfrac{1}{4} H_t(c)^2 + K_g(t) \tag{4.17}$$

where the mean curvature, $H_t(c)$ of $\Sigma_{c,t}$ is given by (4.8). Particularly, the surface $\Sigma_{0,t} \subset \mathcal{B}_t$ has the scalar curvature $K_a(t)$ and

$$K_a(t) = K_g(t) + \kappa_t'(0)^2 \tag{4.18}$$

On the other hand, it follows from (4.5) and (4.13)–(4.16) that the equidistant space $\mathcal{B}_g$ is, at each instant $t \in I$, an Einstein space iff

$$\kappa_t'' = f(\kappa_t, t)$$

$$(\kappa_t(0), \kappa_t'(0)) = (\alpha(t), \beta(t)), \qquad \alpha(t) \neq 0 \tag{4.19}$$

where

$$f(\kappa_t, t) = K_a(t) \exp\{-2[\kappa_t - \alpha(t)]\} \tag{4.20}$$

Since (4.19) is reducible to

$$|\kappa_t'| = [f(\kappa_t, t) + \beta(t)]^{1/2} \tag{4.21}$$

we obtain, comparing (4.18) and (4.21) and taking into account (4.8), that

$$K_g(t) = -\beta(t)^2, \qquad H_a(t) = 2\beta(t), \qquad K_a(t) = 0 \tag{4.22}$$

and it follows from (4.5b), (4.8), (4.14), (4.17), and (4.22) that

$$\kappa_t(X^3) = \tfrac{1}{2} H_a(t) X^3 + \alpha(t) \tag{4.23}$$

Consequently, we obtain from (4.5a) and (4.23) that, for the equidistant Einstein space $\mathcal{B}_g$, the scalar $\Psi_t$ of (3.23) takes the form

$$\Psi_t(X^3) = \exp[-H_a(t) X^3] \tag{4.24}$$

We *conclude* that the (time-dependent) equidistant material space $\mathcal{B}_g$ is an Einstein space (i.e., $\mathcal{B}_g$ is conformally flat of a constant scalar curvature) iff $\mathcal{B}_g$ has a negative scalar curvature dependent on the time parameter $t$

only. In this case each instantaneous material space $\mathcal{B}_t$ is foliated by flat umbilical surfaces with the same constant mean curvature $H_a(t)$. For example, in the case of a distribution of dislocations defined by the condition (3.7), we have [cf.(3.3) and (3.24)]

$$[\mathbf{E}_\alpha, \mathbf{E}_\beta] = 0, \qquad \alpha, \beta = 1, 2 \tag{4.25}$$

The distribution of dislocations so defined is a particular case of the distribution of edge dislocations defined by (2.18), (2.19), and (2.24) if [cf. (3.1), (3.2), (3.22), (4.10), and (4.24)]

$$\mathbf{t} = \mu_g \mathbf{E}_3, \qquad \mu_g = H_a(t) > 0 \tag{4.26}$$

## 5. FINAL REMARKS

Let us consider the distribution of dislocations defined by (3.7) and (4.25). In this case the intrinsic metric tensor **g** can be obtained by means of an isotropic local rescaling of the Euclidean metric (Trzęsowski, 1997) and the time-dependent material space $\mathcal{B}_g$ is, at each instant $t \in I$, an equidistant Riemannian space of a constant negative scalar curvature $K_g(t)$ (Section 4). The material space $\mathcal{B}_g$ is foliated, then, in the equidistant manner (Section 4), by flat umbilical glide surfaces [see the conditions (3.19)–(3.21)] with the same mean curvature $H_a(t)$, $t \in I$, and [see (4.22)]

$$K_g(t) = -\tfrac{1}{4} H_a(t)^2 \tag{5.1}$$

Since, according to (4.10), we have then

$$\rho b_g = \tfrac{1}{2} |H_a(t)| \tag{5.2}$$

we have a distribution of dislocations for which the existence of secondary point defects (Section 2) causes the occurrence of nonplanar glide surfaces. Note that in the dislocation fluid case defined by (2.18), (2.24), and the integrability condition (3.1), the relation (4.10) is not, in general [see (4.26)], valid. Note also that in general, unlike the case (5.1) and (5.2), the flatness of the material space $\mathcal{B}_g$ does not mean a lack of dislocations (Trzęsowski, 1994).

The notions of local glide systems and involutive distributions of local slip planes introduced in this paper characterize, in terms of differential geometry, some aspects of the kinematics of the glide motion of edge dislocation lines. Particularly, they lead to the definition of a class of convective Lagrange coordinate systems distinguished by the condition that, in these coordinate systems, the time-dependent intrinsic metric tensor **g** takes the canonical form (3.23), and specified by some additional conditions [see, e.g., (3.7) and (3.8)]. These convective coordinate systems characterize the dependence of the equidistant property of the Riemannian material space upon the time parameter (Section 4), and may be used for the definition of such material flows within the dislocated crystalline solid that are consistent with its foliation by (virtual) glide surfaces (Trzęsowski, 1998).

## REFERENCES

Aifantis, E. C. (1987). *International Journal of Plasticity*, **3**, 211.

Bilby, B. A., Bullough, R., Gardner, L. R., and Smith, E. (1958). *Proceedings of the Royal Society A*, **244**, 538.

Bullough, R., and Newman, R. (1970). *Reports Progress of Physics*, **33**, 101.

Eisenhart, P. E. (1964). *Riemannian Geometry*, Princeton University Press, Princeton, New Jersey.

Hull, D., and Bacon, D. J. (1984). *Introduction to Dislocations*, Pergamon Press, Oxford.

Ivlev, D. D. (1966). *The Theory of Perfect Plasticity*, Science, Moscow [in Russian].

Kadić, A., and Edelen, D. E. B. (1983). *A Gauge Theory of Disclinations and Dislocations*, Springer-Verlag, Berlin.

Kondo, K. (1955). *RAAG Memoirs*, **1**, 6.

Kröner, E. (1986). *ZAMM*, **66**, T284.

Kröner, E. (1995). *International Journal of Engineering Sciences*, **33**, 2127.

Oding, I. A. (1961). *The Theory of Dislocations in Metals*, PWT, Warsaw [in Polish].

Perzyna, P. (1978). *Thermodynamics of Inelastic Materials*, PWN, Warsaw [in Polish].

Schouten, J. A. (1954). *Ricci-Calculus*, Springer-Verlag, Berlin.

Sikorski, R. (1972). *Introduction to Differential Geometry*, PWN, Warsaw [in Polish].

Sinukov, N. S. (1979). *Geodetic Mappings of Riemannian Spaces*, Science, Moscow [in Russian].

Suzuki, T., Takeuchi, S., and Yoshinaga, H. (1991). *Dislocation Dynamics and Plasticity*, Springer-Verlag, Berlin.

Theodosiu, C. (1976). *International Journal of Engineering Sciences*, **14**, 713.

Thomas, T. Y. (1961). *Plastic Flow and Fracture in Solids*, Academic Press, New York.

Trzęsowski, A. (1993). *Reports on Mathematical Physics*, **32**, 71.

Trzęsowski, A. (1994). *International Journal of Theoretical Physics*, **33**, 931.

Trzęsowski, A. (1997). *International Journal of Theoretical Physics*, **36**, 177.

Trzęsowski, A. (1998). *International Journal of Theoretical Physics*, this issue.

Trzęsowski, A., and Sławianowski, J. J. (1990). *International Journal of Theoretical Physics*, **29**, 1239.

Von Westenholz, C. (1972). *Differential Forms in Mathematical Physics*, North-Holland, Amsterdam.

Yang, W., and Lee, W. B. (1993). *Mesoplasticity and Its Applications*, Springer-Verlag, Berlin.

Yano, K. (1958). *The Theory of Lie Derivatives and Its Application*, North-Holland, Amsterdam.